MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

5 MARCH 2019
9.00 am – 11.00 am
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions excluded the cover page and Appendix. Key formulae are given in the Appendix.

2. Answer **ALL** questions.

3. Write your answers in the answer booklet provided.

4. All necessary working steps must be shown.

Question 1

a. Differentiate the following function with respect to x by using **Chain Rule**.

$$f(x) = 5\ln\left(10x^3 - 5x^2 - 5x\right)$$ (5 marks)

b. Differentiate $27y^2 - 3x^4 + 3x^2y = 3x^4y^3$ by using **implicit differentiation**. (5 marks)

c. Given $f(x) = 4x^3 - 8x^2 + 12$, find the intervals on which the function is increasing and decreasing. (5 marks)

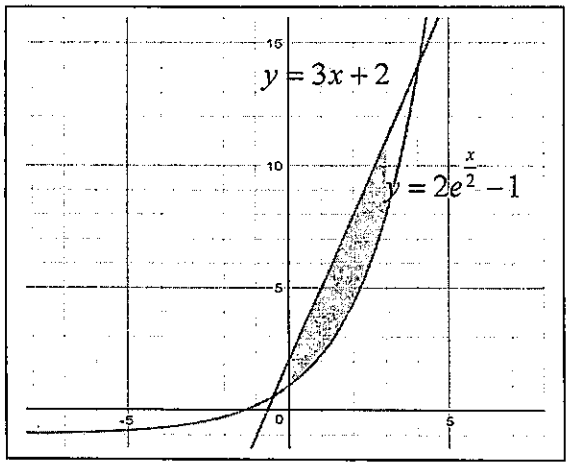
[TOTAL 15 MARKS]

Question 2

a. Use **Substitution Rule** to find $\int 11x\left(7 - 11x^2\right)^{\frac{3}{4}} dx$. (4 marks)

b. Determine $\int (1+10x)e^{5+x}dx$ by using the **Integration by Parts**. (5 marks)

c. The diagram below shows the curve of $y = 2e^{\frac{x}{2}} - 1$ and the straight line of $y = 3x + 2$. Find the area of the shaded region which bounded by these two functions from $x = 0$ to $x = 3$. Round your answer to 2 decimal places. (6 marks)



[TOTAL 15 MARKS]

Continued...

Question 3

a. Solve the differential equation, $\frac{dy}{dx} = \frac{2e^{3x} + 9}{\cos y}$ by using **separable method**.

(2 marks)

b. Identify the $p(x)$ and $q(x)$ of the first order differential equation with non-separable variables, $x\frac{dy}{dx} + 6x^3 y = 7xe^{-7x}$. (2 marks)

c. Find the solution of the differential equation, $\frac{dy}{dx} + p(x)y = q(x)$ given that $p(x) = \frac{2}{x}$ and $q(x) = x^5 + 6$. [Hint: **Use method of integrating factors**]

(3 marks)

d. Find the **general solution** of homogeneous equation, $2y''+15y'-8y = 0$.

(3 marks)

[TOTAL 10 MARKS]

Question 4

a. Given the vertices of a triangle A = (2, -1, 0), B= (-2,1, 3), and C = (1, 0, 1).

i) Determine $\vec{AB}$ and $\vec{AC}$. (2 marks)

ii) Calculate the cross product of $\vec{AB}$ and $\vec{AC}$. (3 marks)

iii) Compute the total area of the triangle. Round your answer to 2 decimal places. (2 marks)

b. If a line passing through the points (3, 2, 3) and (-2, 4, 2), compute the parametric equations of the line. (3 marks)

[TOTAL 10 MARKS]

End of page.

APPENDIX

Derivatives: $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$

Differentiation Rules

General Formulae

1. $\dfrac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$
2. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right]=\dfrac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2}$
3. $\dfrac{d}{dx}(x^n)=nx^{n-1}$
4. $\dfrac{d}{dx}[f(u)]=\dfrac{dy}{du}.\dfrac{du}{dx}$

Exponential and Logarithmic Functions

1. $\dfrac{d}{dx}(e^x)=e^x$
2. $\dfrac{d}{dx}(a^x)=a^x\ln a$
3. $\dfrac{d}{dx}(\ln x)=\dfrac{1}{x}$
4. $\dfrac{d}{dx}(\log_a x)=\dfrac{1}{x\ln a}$

Trigonometric Functions

1. $\dfrac{d}{dx}(\sin x)=\cos x$
2. $\dfrac{d}{dx}(\cos x)=-\sin x$
3. $\dfrac{d}{dx}(\tan x)=\sec^2 x$
4. $\dfrac{d}{dx}(\csc x)=-\csc x\cot x$
5. $\dfrac{d}{dx}(\sec x)=\sec x\tan x$
6. $\dfrac{d}{dx}(\cot x)=-\csc^2 x$

Table of Integrals

1. $\int u\,dv=uv-\int v\,du$
2. $\int u^n\,du=\dfrac{u^{n+1}}{n+1}+C,\quad n\neq -1$
3. $\int\dfrac{du}{u}=\ln|u|+C$
4. $\int e^u du=e^u+C$
5. $\int\sin u\,du=-\cos u+C$
6. $\int\cos u\,du=\sin u+C$
7. $\int\sec^2 u\,du=\tan u+C$
8. $\int\csc^2 u\,du=-\cot u+C$
9. $\int\sec u\tan u\,du=\sec u+C$
10. $\int\csc u\cot u\,du=-\csc u+C$

Application of Integrals:

Areas between Curve, $A=\int_a^b[f(x)-g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$$= \left| \vec{PQ} \times \vec{PR} \right|$$

Area for triangle PQR

$$= \frac{1}{2} \left| \vec{PQ} \times \vec{PR} \right|$$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|} \right)$